Name of College - S-S-College J-Bad

Dept - Mathematics.

TOPIC - Angle of Intersection of two curves.

Class - B-Sc I (HONS)

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Study Material →

If $y = f(x)$ and $y = F(X)$ be the equation of two curves.

The angle between two curves means the Angle between the Tangent drawn at point of intersection of curves.

If α be that angle between them.

then $\tan\alpha = \dfrac{f'(x) - F'(x)}{1 + f'(x) \cdot F'(x)}$

If two curves touches each other

Then $\alpha = 0 \Rightarrow \tan\alpha = 0$

$\Rightarrow f'(x) = F'(x)$

If the curves cut each other at right angle, then $\alpha = 90$

$\Rightarrow \quad 1 + f'(x) \, F'(x) = 0$

$$\Rightarrow \quad 1 + \frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} \cdot \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = 0$$

$$\Rightarrow \quad \frac{\partial f}{\partial y} \cdot \frac{\partial F}{\partial y} + \frac{\partial f}{\partial x} \frac{\partial F}{\partial x} = 0$$

Ex:→ Find the angle of intersection of two curves

$x^2 - y^2 = a^2$ — (I)

$x^2 + y^2 = \sqrt{2} a^2$ — (II)

On adding (I) and (II)

$2x^2 = (\sqrt{2}+1) a^2$

$\Rightarrow x^2 = \frac{(\sqrt{2}+1) a^2}{2} \quad \Rightarrow x = \sqrt{\frac{\sqrt{2}+1}{2}}$

A/s $2y^2 = (\sqrt{2}-1) a^2$

$y^2 = \frac{(\sqrt{2}-1) a^2}{2} \quad y = \sqrt{\frac{\sqrt{2}-1}{2}}$

Now Diff (i) w.r.t respect to x

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y} = f'(x) \text{ (say)}$$

Diff. (ii)

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y} = F'(x) \text{ (say)}$$

$$\therefore \tan\varphi = \frac{f'(x) - F'(x)}{1 + f'(x)\cdot F'(x)}$$

$$= \frac{\frac{x}{y} + \frac{x}{y}}{1 - \frac{x}{y}\cdot\frac{x}{y}} = \frac{\frac{2x}{y}}{1 - x^2/y^2}$$

$$= \left|\frac{2xy}{y^2 - x^2}\right| = \left(\frac{2xy}{x^2 - y^2}\right)$$

Now $xy = \left(\frac{\sqrt{2}+1}{2}\right)^{1/2} a \cdot \left(\frac{\sqrt{2}-1}{2}\right)^{1/2} a$

$$= \frac{a^2}{2}\left[2-1\right]^{1/2} = \frac{a^2}{2}$$

$$x^2 - y^2 = \frac{a^2}{2}\left[(\sqrt{2}+1) - (\sqrt{2}-1)\right] = a^2$$

$\therefore \tan\alpha = \dfrac{2xy}{x^2 - y^2}$

$= \dfrac{2 \cdot a^2}{2 \cdot a^2} = 1$

$\therefore \alpha = 45^0$

Find the condition that the conics

$ax^2 + by^2 = 1$

$a_1x^2 + b_1y^2 = 1$ shall cut orthogonally

Here $f(x) \equiv ax^2 + by^2 - 1 = 0$

$F(x) = a_1x^2 + b_1y^2 - 1 = 0$

The condition under with the two curves or intersect orthogonally is

$1 + f'(x)\, F'(x) = 0$

$f'(x) = -\dfrac{ax}{by}$

$F'(x) = -\dfrac{a_1x}{b_1y}$

$\Rightarrow$

$$1 + \frac{aa_1 x^2}{bb_1 y^2} = 0$$

$\Rightarrow$ $aa_1 x^2 + bb_1 y^2 = 0$

Here x and y are the co-ordinates of the point of intersection of two curves

$ax^2 + by^2 = 1 \Rightarrow aa_1 x^2 + a_1 b y^2 = a_1$

$a_1 x^2 + b_1 y^2 = 1 \quad aa_1 x^2 + ab_1 y^2 = a$

$\therefore$ On substraction.

~~$(a_1 b - ab_1) y^2 = a_1 - a$~~

On substracting

~~$\therefore y^2 = a_1 - a$~~

$(a - a_1)x^2 + (b - b_1)y^2 = 0$

$(a - a_1)x^2 = -(b - b_1)y^2$ and $\frac{x^2}{y^2} = -\frac{b - b_1}{a - a_1}$

Also $\therefore aa_1 x^2 = -bb_1 y^2$

$$\therefore \frac{x^2}{y^2} = -\frac{bb_1}{aa_1}$$

$$\therefore -\frac{b - b_1}{a - a_1} = -\frac{bb_1}{aa_1}$$

$$\Rightarrow \frac{b - b_1}{bb_1} = \frac{a - a_1}{aa_1}$$

$\Rightarrow \frac{1}{b_1} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{a} \Rightarrow \frac{1}{a} - \frac{1}{b}$ This is the condition